DYNAMICAL BEHAVIOR OF WATER- AND GAS-SATURATED POROUS BODIES

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Porous bodies having a fairly rigid skeleton are of particular interest for research since they are encountered rather frequently in practical problems such as soil mechanics. From a study of the behavior of such media certain conclusions can be drawn about the properties of corresponding solid bodies, and the cold compression curve of a solid material in the negative pressure region can be obtained (cf. [1] where a model is proposed which gives a good description of the behavior of porous bodies under shock compression by shock waves with intensities of the order of several megabars). This model does not describe the results of shock compression of porous bodies by shock waves with intensities of the order of kilobars. In addition, it would be desirable to derive the equation of state of a porous medium in order to describe other processes, such as unloading waves. Herrmann [2] proposed a $p-\alpha$ model in which the equation of state is written as two equations, one of which has the usual form of an equation of state, but contains the porosity parameter α, and the other relates the pressure and porosity. This relation was obtained empirically in [2]. Carroll and Holt [3, 4] calculated the dependence of the porosity on pressure theoretically, taking account of the dynamics of pore collapse. In these papers the presence of gas or liquid in the pores was neglected, although the interstitial pressure can play a very important role in considering unloading, and can lead to an effective expansion of the pores after the passage of a shock wave. In the present article we consider the effect of interstitial pressure on the change of porosity.

We treat a porous body as a homogeneous isotropic medium characterized, in contrast with solid bodies, by an additional parameter — the porosity α, which we define as $\alpha = V/V_s$, where V and V_s are respectively the specific volumes of the porous and solid material under identical conditions.

It is required to derive an equation relating the porosity and the components of the stress tensor averaged over a sufficiently large volume, since the actual stresses vary substantially over distances of the order of the distance between pores. To derive this equation we select a cell around a single pore so that the porosity of the cell is the same as that of the body. The shape of the cell is chosen so that the stress tensor is constant on its boundary. Such a cell can always be chosen if the characteristic length of variation of the average stress is much greater than the size of a pore.

A relation must be specified between the state of stress in a cell and the average stress tensor in the porous body. Let $\langle\sigma\rangle_{ij}$ be a component of the average stress tensor, i.e.

$$\langle\sigma\rangle_{ij} = \frac{1}{V}\int\limits_V \sigma_{ij}dV. \tag{1}$$

Various models of the state of stress in a cell can be constructed, assuming, e.g., that

$$\langle\sigma\rangle_{ij} = \sigma_{ij}|_s, \tag{2}$$

where s is the surface of the cell. If there is no gas in the internal pores the Carroll model [3] is obtained.

It would be more consistent in (1) to change from the average over the volume of the body to the average over the equivalent cell:

$$\langle\sigma\rangle_{ij} = \frac{1}{V_1}\int\limits_{V_1} \sigma_{ij}dV,$$

where V_1 is the volume of a cell. Using the identity

$$\sigma_{ij} = \sigma_{ik}\frac{\partial x_j}{\partial x_k} = \frac{\partial}{\partial x_k}(x_j\sigma_{ik}) - x_j\frac{\partial \sigma_{ik}}{\partial x_k}$$

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and the equation of motion of the material

$$\rho \frac{dv_i}{dt} = \frac{\partial \sigma_{ik}}{\partial x_k},$$

we obtain finally instead of Eq. (2)

$$\langle \sigma \rangle_{ij} = \sigma_{ij}|_s - \frac{1}{V_1} \int\limits_{V_1} \rho x_j \frac{dv_i}{dt} dV, \tag{3}$$

where v_i is a velocity component, ρ is the density of the solid medium, and $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the Lagrangian derivative.

Equation (3) gives a relation between the average values of the stresses in a solid body and the characteristics of the process of pore collapse, since by integrating the equation of motion v can be found in terms of the porosity and its derivatives.

Calculations show that under pressures much smaller than the elastic constants of the solid material (tens of kilobars) the main compression occurs as a result of pore collapse, and consequently the solid material can be assumed incompressible, and the pressure of the material in a pore can be calculated in the adiabatic approximation.

We assume that in the range of pressures considered a solid material is an incompressible elastoplastic medium which satisfies the Mises condition in the plastic phase.

If the width of the loading wave front is much greater than the distance between pores, the behavior of a porous body can be described by considering a single cell in the field of a uniform wave varying only with time whose amplitude p(t) determines the components of the average stress tensor $\langle\sigma\rangle_{ij} = -p(t)\delta_{ij}$. The equivalent cell in this case, i.e., for hydrostatic compression, is a hollow sphere with an outside radius b and an inside radius a such that a is the average size of a pore, and hence $b^3/(b^3 - a^3) = \alpha$ is the porosity. The pressure in a pore is $q(\alpha)$, and $\sigma_{ij}|_{t=0} = -q(\alpha_0)\delta_{ij}$. As will be shown later, the main compression occurs for complete plasticity of the whole cell, and consequently the initial stress distribution is unimportant. The process is described by the equation

$$\rho \frac{dv}{dt} = \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\varphi)}{r}.$$

The yield condition is $|\sigma_r - \sigma_\varphi| = Y$; the boundary conditions are $\sigma_r|_{r=a} = -q(\alpha)$ and $\sigma_r|_{r=b} = \sigma_n|_s$; the latter is calculated from Eqs. (2) or (3); from (2) $\sigma_r|_{r=b} = -p(t)$. By using (3) we have

$$\sigma_r|_{r=b} = -p(t) + \frac{3}{b^3} \int\limits_a^b \rho r^3 \frac{dv}{dt} dr.$$

Integrating the equation of motion we obtain

$$\tau^2 Y Q(\ddot{\alpha}, \dot{\alpha}, \alpha) = p(t) - q(\alpha) - p_{eq}(\alpha), \tag{4}$$

where

$$p_{eq}(\alpha) = \int\limits_a^b \frac{2(\sigma_r - \sigma_\varphi)}{r} dr; \quad \tau^2 = \frac{\rho a_0^2}{3Y(\alpha_0 - 1)^{2/3}};$$

$$\tau^2 Y Q(\ddot{\alpha}, \dot{\alpha}, \alpha) = \frac{3}{b^3} \int\limits_a^b \rho r^3 \frac{dv}{dt} dr - \int\limits_a^b \rho \frac{dv}{dt} dr.$$

Using the incompressibility condition we calculate the mass velocity $v = \dot{\alpha} a_0^3 / 3(\alpha_0 - 1) r^2$ and $Q(\ddot{\alpha}, \dot{\alpha}, \alpha)$. By using (2)

$$Q(\ddot{\alpha}, \dot{\alpha}, \alpha) = -\ddot{\alpha}[(\alpha - 1)^{-1/3} - \alpha^{-1/3}] + \frac{\dot{\alpha}^2}{6}[(\alpha - 1)^{-4/3} - \alpha^{-4/3}],$$

and by using (3) we have

$$Q(\ddot{\alpha}, \dot{\alpha}, \alpha) = -\ddot{\alpha}\left\{\frac{3}{2}[(\alpha - 1)^{-1/3} - \alpha^{-1/3}] - \frac{(\alpha - 1)^{-1/3}}{2\alpha}\right\} + \frac{\dot{\alpha}^2}{6}\left\{4\frac{(\alpha - 1)^{-4/3}}{\alpha} - 3[(\alpha - 1)^{-4/3} - \alpha^{-4/3}]\right\}, \tag{5}$$

where $\ddot{\alpha}$ and $\dot{\alpha}$ are time derivatives.

The interstitial pressure $q(\alpha)$ can be found in the adiabatic approximation

$$q(\alpha) = \begin{cases} q_0\left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^\gamma & \text{for gas,} \\ q_0\left[\left(\frac{\alpha_0 - 1}{\alpha - 1}\right)^\gamma - 1\right] & \text{for water.} \end{cases}$$

The expression for $p_{eq}(\alpha)$ as a function of the state of the material has the form

$$p_{eq}(\alpha) = \begin{cases} \frac{4G}{3}\frac{\alpha_0 - \alpha}{\alpha(\alpha - 1)}, & \alpha_1 < \alpha < \alpha_0, \\ \frac{2}{3}Y\ln\frac{2Ge(\alpha_0 - \alpha_1)}{Y(\alpha - 1)} + \frac{4G(\alpha_0 - \alpha)}{3\alpha}, & \alpha_2 < \alpha < \alpha_1, \\ \frac{2}{3}Y\ln\frac{\alpha}{\alpha - 1}, & 1 < \alpha < \alpha_2. \end{cases}$$

In the range $(2G\alpha_0 + Y)/(2G + Y) = \alpha_1 < \alpha < \alpha_0$ the material behaves elastically; for $(2G\alpha_0)/(2G + Y) = \alpha_2 < \alpha < \alpha_1$ the material is partially plastic (a plastic front is propagated from a pore); and for $1 < \alpha < \alpha_2$ the material behaves like a plastic.

We note that the change of porosity is small in the first two phases of compression (the elastic and elasto-plastic phases):

$$(\alpha_0 - \alpha_2)/\alpha_0 = Y/(2G + Y) \ll 1.$$

Considering unloading with $\alpha = \alpha_-$, i.e., expansion of a pore (we assume that complete plasticity was reached in compression) we obtain an equation analogous to Eq. (4):

$$\tau^2 YQ(\ddot{\alpha}, \dot{\alpha}, \alpha) = p(t) + p_{eq}(\alpha) - q(\alpha),$$

where τ, Q, and q are the same as before, and the function p_{eq} is found from

$$p_{eq}(\alpha) = \begin{cases} \frac{4G(\alpha - \alpha_-)}{3\alpha(\alpha - 1)} - \frac{2}{3}Y\ln\frac{\alpha}{\alpha - 1}, & \alpha_- < \alpha < \alpha_3 = \frac{G\alpha_- - Y}{G - Y}, \\ \frac{2}{3}Y\ln\frac{G^2(\alpha - \alpha_-)^2 e^2}{Y^2(\alpha - 1)\alpha} - \frac{4G}{3}\frac{\alpha - \alpha_-}{\alpha}, & \alpha_3 < \alpha < \alpha_4 = \frac{G\alpha_-}{G - Y}, \\ \frac{2}{3}Y\ln\frac{\alpha}{\alpha - 1}, & \alpha > \alpha_4, \end{cases}$$

with the process passing through the same three phases as in compression. Just as in the expansion of pores the change in porosity is small in the first two phases:

$$(\alpha_4 - \alpha_-)/\alpha_- = Y/(G - Y) \ll 1,$$

so the change of porosity can be neglected in the first two phases, and we can investigate the equation

$$\tau^2 YQ(\ddot{\alpha}, \dot{\alpha}, \alpha) = p(t) - q(\alpha) \pm \frac{2}{3}Y\ln\frac{\alpha}{\alpha - 1}, \tag{6}$$

where the minus sign is for pore collapse and the plus sign for expansion.

Equation (6) with Q given by (5) was integrated numerically using values characteristic for porous aluminum: $\rho = 2.7$ g/cm^3, Y = 3 kbar. The initial porosity was taken as $\alpha_0 = 1.4$ and the pore size as $a = 0.2$ mm. The interstitial pressure was specified in the form of a gas adiabat with an exponent $\gamma = 1.4$.

Calculations were performed for various initial interstitial pressures and various amplitudes and durations of a rectangular pressure pulse (the pulse amplitude and duration were chosen so that $p\Delta t = Y\tau$).

Figure 1 and Table 1 show the time variation of porosity for various pressure pulses with an initial interstitial pressure $q_0 = 3.33 \cdot 10^{-4}$ Y. Figure 2 and Table 2 show similar results for an initial interstitial pressure $q_0 = 3.33 \cdot 10^{-2}$ Y.

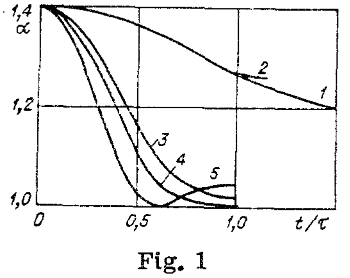

Fig. 1

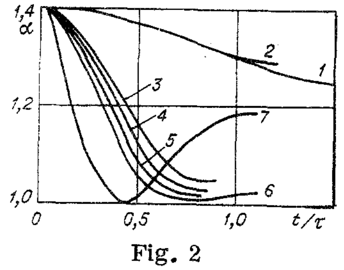

Fig. 2

TABLE 1

No. of curve	Pulse amplitude	Pulse duration	Minimum porosity	Final porosity
1	1	∞	1,191	1,191
2	1	1	1,262	1,262
3	2	0,5	1,023	1,023
4	2,33	0,429	1,002	1,002
5	3,33	0,3	1 0000002	1,042

TABLE 2

No. of curve	Pulse amplitude	Pulse duration	Minimum porosity	Final porosity
1	1	∞	1,239	1,239
2	1	1	1,291	1,291
3	2	0,5	1,048	1,048
4	2,33	0,429	1,028	1,028
5	2,67	0,375	1,017	1,018
6	3	0,33	1,010	1,028
7	10	0,1	1,001	1,181

Figures 1 and 2 show that the presence of gas in the pores can lead to a dependence of the final porosity on the pulse amplitude and duration which is far from monotonic. Thus, as the pulse amplitude is increased the inertia of the process of change of porosity begins to manifest itself: In the loading phase a pore passes through the equilibrium position corresponding to the given pulse and achieves an intense backward motion. The limiting porosity α_+ at which backward motion begins can be found from the equation

$$q(\alpha_+) = \frac{2}{3} Y \ln \frac{\alpha_+}{\alpha_+ - 1}$$

and for $q_0 = 3.33 \cdot 10^{-2}$ Y is equal to 1.017.

The nonmonotonic nature of the final porosity considered above with the passage of an intense loading wave can lead to a nonmonotonic dependence of the porosity of soil close to the source of a shock wave on the distance from the source. The porosity close to and far from a source can be larger than in an intermediate region, but quantitative data can be obtained only for a specific calculation of such a problem.

LITERATURE CITED

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